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ABSTRACT

A study was undertaken of the effects of formal education and conservation of numerousness on addition and subtraction problem types. Thirty-six kindergarten and 36 first-grade subjects randomly selected from one area of a school district were administered measures of conservation, problem-solving success, and modeling ability. Following factor analysis of the instruments, and a regression analysis to ascertain demographic effects, a nested posttest only control group design was analyzed using a covariate MANOVA technique. Both formal schooling and conservation significantly affected the subject's modeling and accuracy scores (p less than .05). Further, transformational addition and take-away subtraction were significantly (p less than .05) more difficult than other problem types. (Author/SD)

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An Analysis of Kindergarten and First Grade Children's Addition and Subtraction Problem Solving Modeling and Accuracy

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SE 020 802

An Analysis of Kindergarten and First Grade Children's Addition and Subtraction Problem Solving Success

Children's mathematics problem solving abilities and strategies have been critically examined by many scholars. They have documented the effects of reading skills (Vanderlinde, 1964), problem structure (Burns and Yonally, 1964), familiarity with problem operants (Steffe, 1967), and degree of involvement of the student (Chase, 1960) on the problem solving abilities of students. Other researchers have established that students' problem solving abilities may be enhanced when models, correction techniques, intermittent praise, graduated difficulty problems, drawings, and peer problems are employed (Riedesel and Burns, 1973). Few, however, have dealt with the effects of the developmental construct of conservation upon mathematical reasoning ability.

The variable of conservation of numerousness was introduced to the mathematics education community from the translated writings of Jean Piaget (1965), the text by Flavell (1963), and the research of Elkind (1961), Dodwell (1960) and Wohlwill (1962). Research studies related to this variable began with those by Van Engen and Steffe (1966) on first grade addition problem solving skills, LeBlanc (1968) on subtraction problem solving skills, Steffe (1970) on addition, and Steffe and Johnson (1971) on addition and subtraction. In these studies, the youngsters'

Abilities to make quantitative comparisons (either gross, intensive or extensive) was treated as an independent variable in studying problem solving abilities.

Previous researchers have reported significant differences between the problem solving achievement of first graders who conserve and those who do not conserve. These researchers also found significant differences in problem solving achievement related to problem characteristics. In addition, it was found that problems which involved a transformation* were significantly more difficult than those which did not involve a transformation. In subtraction, the "take away" subtraction problems without transformations were found to be significantly more difficult than "take-away" subtraction problems with transformations.

While previous studies were carefully conducted and subjected to rigorous statistical analysis, a major question remains unanswered. Since these addition and subtraction studies used a population completing the first grade, are the "transformation" and "take-away" findings related to the developmental level of the subjects or are they the result of first grade instruction? This is a particularly cogent question when one analyzes the present state of first grade instruction and realizes that most first grade teachers model addition as a process which involves transformational take-away process.

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*A transformation is defined as an implied physical movement of objects associated with an addition problem describing joining of sets, e.g., "Mary has three dolls and her mother gave her two more." No transformation is a static condition, e.g., "John has two frogs and David has three frogs."

The Conflict. As Piaget (1965) noted, and as one can readily observe with children who are in kindergarten and grade one, five does not always equal five. Young children are so bound by perception that they believe numerical changes occur which do not, but which appear as though they do. For example, given five coins arranged as in figure 1, the young child believes the coins on the left are "more". However, if

Insert Figure I About Here

the dispersion is equal so that the perception is similar, i.e., the space occupied looks the same, the child will think that neither is more or less than the other.

In the study by Van Engen and Steffe (1966), one can see why, on the basis of the above, children thought 2 + 3 > 5 as noted in Figure 2. On the basis of this established perceptual

Insert Figure 2 About Here

confusion, it would appear that understanding of addition and equality would best be served if addition could be taught without transformations (movement). Thus, 2 + 3 would "look like" and, hence, be thought to be another name for 5. As Underhill (1972) states, "The number sentence 2 + 3 + 5 and the number sentence 2 + 3 + 4 + 1 indicates that the number which is named in each instance is the same number. In mathematics the equal sign means names the same number as." Certainly an intuitive concept of equals cannot be developed in non-conservers by use of the addition concept with transformations. However, a difficulty

Figure 1. Illustration of Perceptual Problem

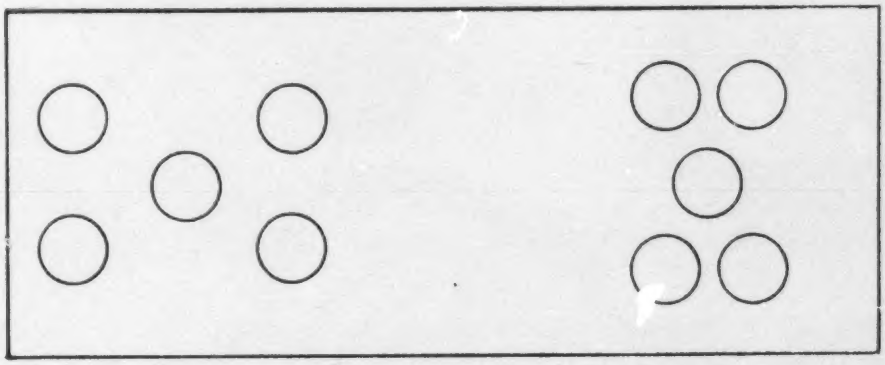
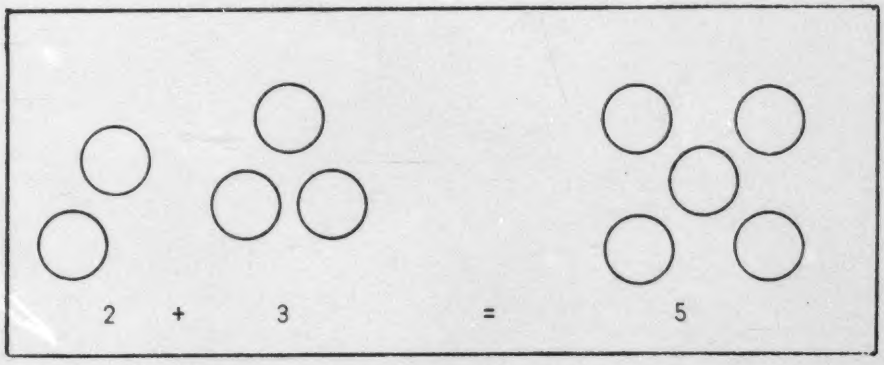


Figure 2. Illustration of Perceptual Confusion



exists because Steffe (1967) and LeBlanc (1968) found that preconservers' addition and subtraction problem solving successes were significantly better in problems involving transformations.

The purposes of this research effort were (1) to study the influences of formal instruction on addition and subtraction problem solving behaviors of young children, and (2) to study further the relations among two types of addition and three types of subtraction problem solving success, modeling behaviors, and abilities to make quantitative comparisons.

Addition and Subtraction

In this section, the results are summarized and related to previous research. Certain conjectures are made concerning directions for future study.

Insert Table 1 About Here

As noted in Table 1, non-transformational additional addition was easiest for both grade levels to model. Comparison was the most difficult problem type to model. While first graders were most successful on both types of addition and take-away subtraction. Comparison subtraction was the most difficult problem type for both grade levels.

Insert Figure 3 About Here

The ability of kindergarten and first grade children to model transformational addition was significantly effected by their conservation abilities as measured by the TQC ($P = .03$), but there was no significant effect upon non-transformational modeling by quantitative comparison abilities ($P = .63$). Children with high TQC scores modeled better than children with low TQC scores. Also, there was a significant effect on schooling on

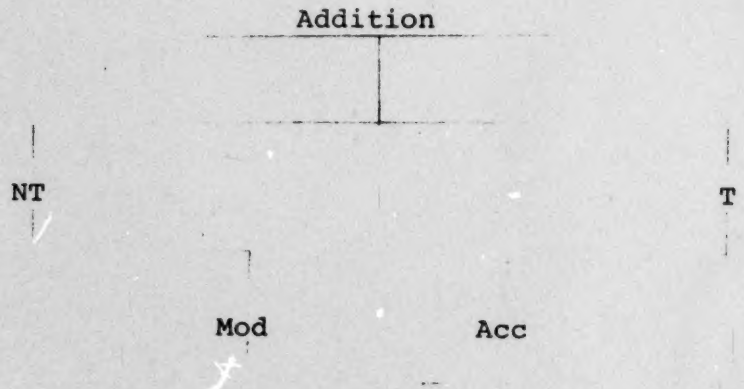
Table 1

Modeling and Accuracy Means by Problem Type and Schooling

	Addition		Subtraction		
Modeling*	Transf.	Non-Tr.	Take Away	Comparative	Additive
K	3.17	3.67	3.14	2.19	2.50
1	3.22	3.94	3.14	2.89	3.22
Accuracy**					
K	.92	1.17	1.50	.25	.61
1	1.67	1.75	1.67	.78	1.08

*Maximum score possible: 4
**Maximum score possible: 2

Fig. 6 Summary of the Analysis of Covariance of the Addition Scores



* $P < .05$

non-transformational modeling abilities (P = .06), but not on transformational modeling abilities (P = .59); the significant effect may be spurious, as noted earlier.

Addition accuracy was significantly effected by schooling (Transformational, P = .004; Non-Transformational, P = .004). First graders had higher success rates than kindergarten children. Again, TQC scores significantly effected accuracy scores (Transformational, P = .002; Non-Transformational, (P = .008). Subjects with high TQC scores had high success rates.

Accuracy and Previous Studies As in the Steffe (1967), and Steffe & Johnson (1970) studies, conservation was related to transformational and non-transformational addition accuracy ($p < .01$). All three studies support a significant relation between addition problem solving ability and learners' performances on quantitative comparisons tasks. Also, when the effects of conservation and age are removed, formal education (schooling) is significantly related to non-transformational addition accuracy ($p < .01$). This lends support to the initial conjecture that results of the Steffe study were influenced by training, but the effects of schooling are not strong enough to wipe out differences which might otherwise exist, as the differences in accuracy performances were consistent across kindergarten and first grade subjects.

Results of the present study are in agreement with the Steffe and Johnson study and in conflict with the Steffe study relative to the difficulty of transformational and non-transformational addition problems; based on these three studies it is a fairly safe conclusion that these problem types are of similar difficulty

for first grade children when instruction is consistent with good modeling behaviors. One factor which may influence accuracy of young learners is the required use of manipulatives as in the present study. Alternate roles of manipulative usage is an area in need of further investigation.

It is very interesting to note that kindergarten and first grade children's abilities to model addition problems differ in variables which influence performance. Their abilities to model transformational addition are significantly related to quantitative comparisons ($p = .03$) but not formal schooling ($p = .591$). However, just the opposite is true for non-transformational addition; it is not related to quantitative comparisons ($p = .626$) but approaches significance on schooling at the .05 level ($p = .057$). These results indicate that first grade schooling enhances children's performances on non-transformational modeling which in turn tends to wipe out differences by the end of the first grade, and growth in quantitative comparisons abilities during the first grade tends to wipe out differences on modeling which favored transformational addition. Thus, the growth in children's modeling abilities in addition appear to be due to combined effects of maturation and schooling.

It would appear that more conscientious training efforts on non-transformational modeling in the first grade would eliminate modeling differences among young learners. Additional research is needed relative to the effects of specific non-transformational and transformational modeling and accuracy training. Present instruction tends to be biased in favor of transformational approaches. As was mentioned at the beginning of this report, such an instructional approach raises serious questions since the efficacy of such

experiences is influenced by the Van Engen and Steffe perceptual da

Insert Figure 4 about here

The ability to model take-away subtraction and additive subtraction was not significantly effected by schooling ($p = .36$ & $.11$) but the a bility to model comparison subtraction was significantly effected by schooling ($P = .02$). First grade subjects had higher comparison subtraction modeling scores than kindergarten subjects. Also, while the ability to make quantitati comparisons had no significant effect upon subjects' take-away modeling performance ($P = .78$), it did have a highly significant effect upon additive ($P = .005$) and comparison modeling scores ($p = .005$ & $.02$). Subjects with high TQC scores had high additive and comparison subtraction modeling scores.

Moving from modeling to accuracy, the subjects' take away and comparison subtraction accuracy was not significantly effected by schooling ($p = .08$ & $.33$) but additive subtraction accuracy was ($P = .01$). First grade subjects had higher accuracy scores on additive subtraction than kindergarten subjects. Also, the abilit to make quantitative comparisons had no significant effect on take-away and comparison accuracy ($P = .47$ & $.06$), but it did have a significant effect on additive subtraction accuracy ($P = .000$). Subjects with high TQC scores had high additive subtraction modeling scores.

A very interesting result was the lack of effects of conservation and schooling (with variance due to age removed) on take away modeling and accuracy. Even though take-away subtraction mea were significantly higher than comparison and additive on both modeling and accuracy, the two variables grade level and conservati

Fig. 7 Summary of the Analysis of Covariance of the Subtraction Scores

Subtraction													
Add				Comp				TA					
Acc		Mod		Acc		Mod		Acc		Mod			
Cons	Sch	Cons	Sch	Cons	Sch	Cons	Sch	Cons	Sch	Cons	Sch		
.000*	.011*	.005*	.107	.059	.335	.020*	.021*	.470	.078	.783	.359		

* P < .05

did not account for differential performances among subjects on their take-away behaviors. Other variables must be explored if such differences are to be meaningfully explained.

Take-Away Subtraction and Previous Studies. Differences among the three studies (LeBlanc, Steffe, and Johnson, the present study) center about two issues: (1) transformations, and (2) quantitative comparisons. The initial conjecture that LeBlanc's (1968) results were inflated due to first grade instruction was not supported by the study results.

While LeBlanc found a significant effect on accuracy ($p \leq .01$) related to transformations, Steffe and Johnson (1970) inferred they did not (no statistic reported). In the present study only transformational take-away problems were used. It is our view that there is probably no difference. For, in LeBlanc's description of tasks he indicated that the researcher manipulated the objects. This form of transformational task is more readily solved by the subject since a subset is physically removed by the researcher while there appeared to be no visual distinction among the subsets of the non-transformational sets.

In the case of quantitative comparisons, LeBlanc found a significant difference related to accuracy ($P \leq .01$) and Steffe and Johnson found none. In the present study no significant difference was found ($P = .470$). These differences are probably attributable to the operation of certain testing biases alluded to in the previous paragraph; it seems safe to conclude that quantitative comparisons skills do not significantly effect either modeling or accuracy take-away subtraction performances of young

learners. If any such differences do exist, they are almost certainly due to differences on non-transformational subtraction.

The precise role of manipulatives usage is a question worth further pursuit also, since LeBlanc had the researcher manipulate Steffe and Johnson made the use of manipulatives optional with subjects, and subjects were required to use manipulatives in the present study. This, too, may be an important factor in accounting for differing research outcomes.

Additive Subtraction and a Previous Study. Steffe and Johnson found no significant relationship between quantitative comparison abilities on accuracy and modeling when manipulatives usage was required with transformational items. Thus, the differences can probably be attributed to differential usage of manipulatives and/or differential treatment of transformational items by researchers; Steffe and Johnson (1970), however, felt that differences were not attributable to transformations. Additional research is needed to clarify these issues. According to Howlett's (1973) research, inclusion may be a basis for formulating other alternate hypotheses designed to explain differential performances on this problem type.

Procedures and Materials

Thirty-six kindergarten and thirty-six first grade children were randomly selected from one Area of the Houston Independent School District of Houston, Texas. Because of its geographic distribution, cutting across the urban and suburban sections of the city, it was felt that the Area contained a representative cross-section of ethnic and socio-economic groups. There were 2,341 kindergarten and 3,061 first grade children in the Area.

The Test of Quantitative Comparisons (TQC), developed by Harper and Steffe (1968), and the Houston Addition and Subtraction Problem Solving Test (HASPST), developed by Underhill and Shores (1975), were individually administered to the kindergarten subjects in March 1974 and to the first grade subjects in May 1974. These instruments provided individual measures of conservation, problem solving success, and problem solving modeling behaviors.

All tests were administered at the respective elementary schools attended by the subjects. Testing was done in a non-classroom and no more than four persons were present at the time of testing: two subjects and two data collectors. The administration "time" for the TQC was approximately 18-20 minutes; the time spent with each subject was about 30-35 minutes. Since there was considerable novelty and involvement, subjects fatique and restlessness appeared to be minimal. Nearly all subjects appeared to enjoy the individual attention and to be quite interested throughout.

The TQC consisted of 20 items: four warm-up items and 16 evaluated responses. The 20 items were adminstered in the same order to all 72 subjects as described by Steffe and Harper (1968). Some TQC items involve static set comparisons and other items involve physical transformations. From each subject's responses a measure related to conservation of number was obtained.

The HASPST consisted of 11 items: 6 subtraction (2 each of take-away, comparison and additive), 4 addition (2 each of transformation and non-transformation), and one transition item. Before each individual administration, the order, addition then subtraction or vice versa, was determined by the flip of a coin. The order of the six subtraction questions and four addition questions was also determined randomly for each administration. The transition item was included because a pilot study revealed that responses to the first item in a sequence at the point of operation change was often characterized by a continuation of the first operation; this item always agreed in problem type with the last randomly ordered problem in the ten evaluated items; performance on the transition item was not used in the analysis.

The subject and researcher, as before, were usually seated across from one another at a table. In front of the subject was a small basket of chips and two eight inch, cardboard dolls. To provide for maximal success of the subjects, the subject named the dolls, e.g., Michelle and Ramos, and the chips, e.g., hamburgers. All eleven questions then used the names and label provided by each subject. Each question was "chunked", i.e., presented one-sentence-at-a-time with a pause for the subject to "act out" the

sentence with the manipulatives provided: subjects' use of manipulatives was required and probing questions were used to obtain responses not spontaneously provided or unclear to the researcher. Each subject received 0 or 1 point for the accuracy of each response (the criterion was a correct response ± 1). Each subject received 0,1, or 2 points for the modeling behaviors exhibited relative to each problem according to criteria specified for each of the five problem types (see Underhill and Shores, 1975).

Design

The effects of two independent variables related to the development of the child were of primary interest. Ability to conserve numerousness has been established as a function of the psychological development of the child (Piaget, 1965). Schooling is experiences socially ascribed to children of specific age levels and, as such, constitutes a diversion of development. Neither of these variables is under sufficient experimental control to be manipulated. Thus, to treat these variables as independent, the researchers examined longitudinal and pre-experimental design alternatives. It was determined that the stability of the school district's population and the consistency of instruction received by first grade students was sufficient, in light of the probable expense and subject mortality in a longitudinal study, to justify a post test only control group design with an implicit dual treatment of one year of schooling and one year of psychological development of conservation abilities.

It was necessary to account for the nesting of variables

within each state (accuracy and modeling) of the dependent variable of problem solving, for problem solving occured in addition and subtraction cases with two and three types of problems in each case, respectively. The data, thus, took the form presented in figure 5. With the nested design, it

Insert Figure 5 About Here

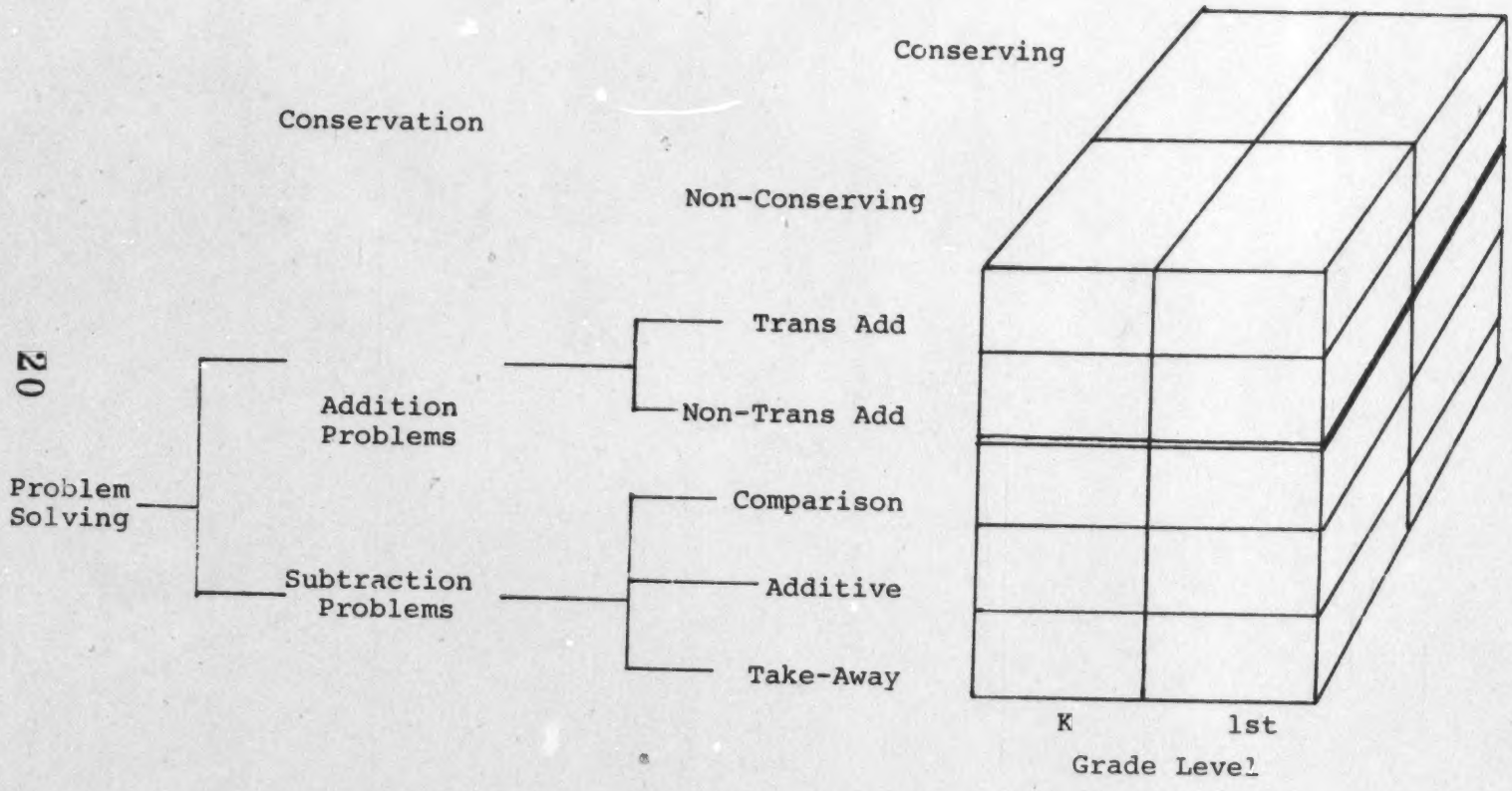
was possible to trace significant main effect findings to the specific type of problem which contributed to the main effect finding.

Analysis

Prior to testing the independent variables, it was necessary to ascertain the interpretability of the measures. A principle component factor analysis of the Addition-Subtraction Problem Solving Test (Underhill and Shores, 1975) revealed that subjects responded consistently to combined addition problems and the three distinct subtraction types when accuracy scores were used. However, when modeling scores were analyzed, the additive subtraction items combined with one type of addition.

Initial multiple linear regression analysis of the dependent variables using sex and age (in months) as criterion variables confirmed that variance due to age must be statistically removed from subsequent analyses due to its relationship to both "schooling" and "Conservation". Likewise, a systematic, although

FIG. 5 Data Utilized in the Study



non-significant sex bias was statistically controlled to eliminate possible contamination of unique cells during the analyses.

The nested analyses of covariance of the modeling scores are summarized in figure 6 and tables. The subjects' abilities

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Insert Figure 6 About Here

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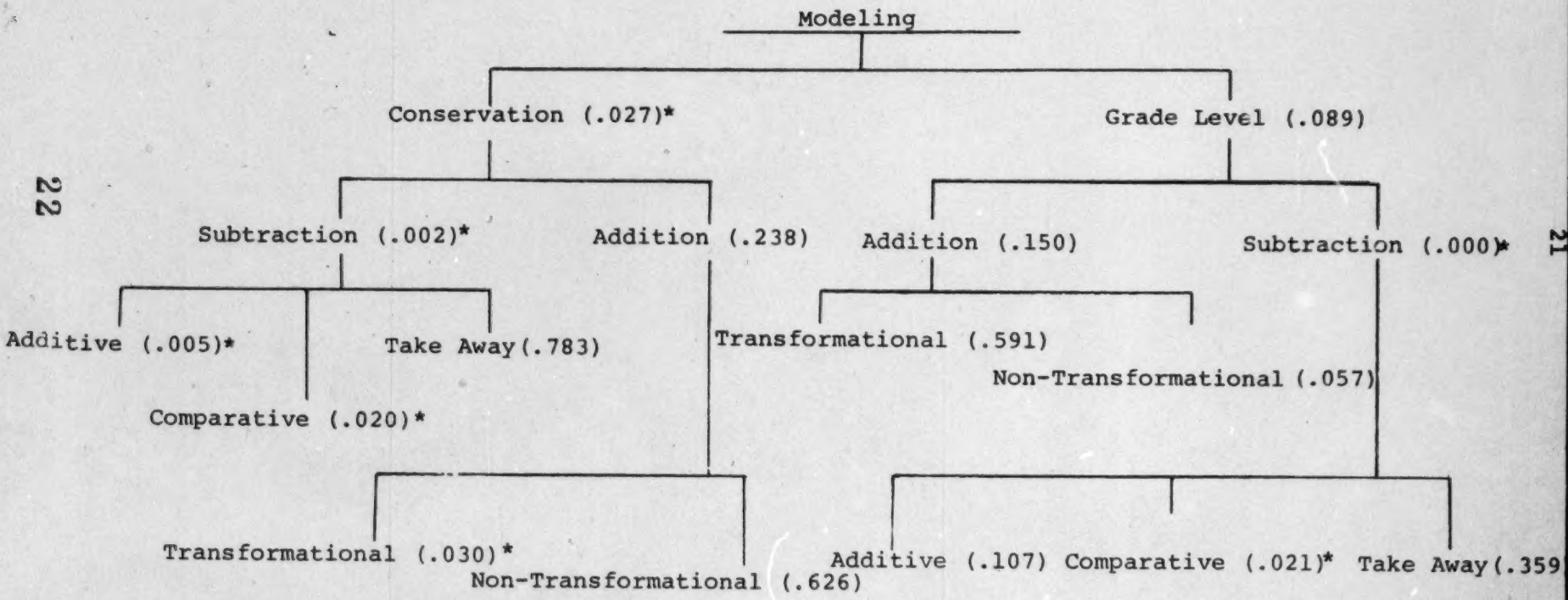
to model arithmetical operations was significantly affected by their abilities to conserve numerousness ($p = .027$), but the effect of prior schooling upon their modeling scores was only significant at the $p = .089$ level. It must be remembered that the subjects' ages (in months) and sexes were treated as covariates in the analyses.

Insert Tables 2-4 About Here

The effect of conservation upon the modeling of arithmetic operations was attributable to subjects' modeling of subtraction problems ($p = .002$). More specifically, the major effect was traced to significant differences due to conservation of numerousness upon additive ($p = .02$) subtraction scores. It is noted that the significant effect of conservation upon transformational addition scores ($p = .03$) was not sufficiently strong to appear as a main effect.

Within the reported effects of schooling upon the subjects' ability to model arithmetical operations it should be noted that while schooling significantly effects the subjects' ability

Fig. 6 Summary of Analysis of Covariance of the Modeling Scores



* P < .05

Table 2

Summary of Analysis of Covariance Based on Total Modeling Scores for Grade Level and Conservation

I.

Source	% Var Percent of Variance	SS	df	MS	F	Sig.Level
Covariates	5.26	51.7302	2			
Grade Level	8.15	80.2222	1	80.222	2.98	.089
Conservation	12.45	122.5552	1	122.552	5.09	.027
Error	74.14	729.4906	67	11.37149		
Total	100.00	984.0000	71			

II. Table of Means

Scores	Kindergarten K	First Grade 1st	Non Conserves NC rs	Conserves C rs
$\bar{X}$ Modeling	14.61	16.72	14.31	16.63

Table 3

Summary of Analysis of Covariance Based on Addition and Subtraction Modeling Scores for Grade Level and Conservation

I Addition:

Source	% Var Percent of Variance	SS	df	MS	F	Sig. Level
Covariates	2.96	3.3781	2			
Grade Level	2.81	3.5556	1	3.5556	2.13	.150
Conservation	3.35	5.2855	1	5.2855	1.42	.238
Error	92.26	145.5586	67	12.6815		
Total	100.00	157.7778	71			

II Subtraction:

Source	% Var Percent of Variance	SS	df	MS	F	Sig.Level
Covariates	5.55	34.2065	2			
Grade Level	8.11	50.0000	1	50.0000	10.16	.002
Conservation	12.48	76.9381	1	76.9381	16.35	.000
Error	73.86	455.2998	67	7.0218		
Total	100.00	616.444	71			

III Table of Means

Scores	K	1st	NC	C
$\bar{X}$ Modeling, Additive	6.72	7.17	6.81	7.02
$\bar{X}$ Modeling; Subtraction	7.89	9.56	9.41	7.50

Table 4

Summary of Analysis of Covariance Based Upon Addition and Subtraction Modeling Subscores for Grade Level and Conservation

I Transformation Addition:

Source	% Var	SS	df	MS	F	Sig. Level
Covariates	.00	.2431	2			
Grade Level	.00	.2222	1	.2222	.292	.591
Conservation	7.31	8.0703	1	8.0703	4.927	.030
Error	92.46	102.1210	67	1.5346		
Total	100.10	110.4444	71			

II Non-Transformation Addition:

Source	% Var	SS	df	MS	F	Sig. Level
Covariates	5.46	1.662	2			
Grade Level	3.73	1.3889	1	1.3889	3.752	.057
Conservation	.00	.0001	1	.0001	.240	.625
Error	91.82	34.2268	67	.42421		
Total	100.00	37.2778	71			

III Additive Subtraction:

Source	% Var	SS	df	MS	F	Sig. Level
Covariates	3.03	4.6918	2			
Grade Level	6.07	9.3889	1	9.3889	2.668	.107
Conservation	16.17	25.0075	1	25.0075	8.537	.005
Error	74.72	115.5229	67	1.7594		
Total	100.00	154.6111	71			

Table 4 (con't)

IV
Comparison
Subtraction:

Source	%Var	SS	df	MS	F	Sig.Level
Covariates	18.32	12.2518	2			
Grade Level	13.82	9.3889	1	9.3889	5.614	.021
Conservation	16.05	10.9034	1	10.9039	5.644	.020
Error	52.10	35.3998	67	.6899		
Total	100.00	67.9444	71			

V
Take Away
Subtraction

Source	% Var	SS	df	MS	F	Sig.Level
Covariates	.01	1.6158	2			
Grade Level	1.45	2.3472	1	2.3472	.853	.359
Conservation	.00	.0248	1	.0248	.077	.783
Error	97.55	157.9983	67	2.3036		
Total	100.00	161.9861	71			

VI

Table of Means

Scores	K	1st	NC	C
$\bar{X}$ Modeling Tr.Add	3.17	3.28	2.96	3.37
$\bar{X}$ Modeling NTV.Add	3.67	3.94	3.85	3.78
$\bar{X}$ Modeling Add Sub	2.50	3.22	2.38	3.13
$\bar{X}$ Modeling Comp Sub	2.17	2.89	2.08	2.78
$\bar{X}$ Modeling TA Sub	3.08	3.44	3.04	3.04

to model subtraction processes (p = .00), this effect was not strong enough to be reflected in the main effect of schooling. It should also be noted that comparative subtraction is the subtraction modeling process most effected by schooling (p = .021) and transformational addition is the addition process most effected by schooling (p = .057).

Insert Figure 7 About Here

The nested analyses of covariance of the accuracy scores are summarized in figure 5 and tables 4-6. The ability of the

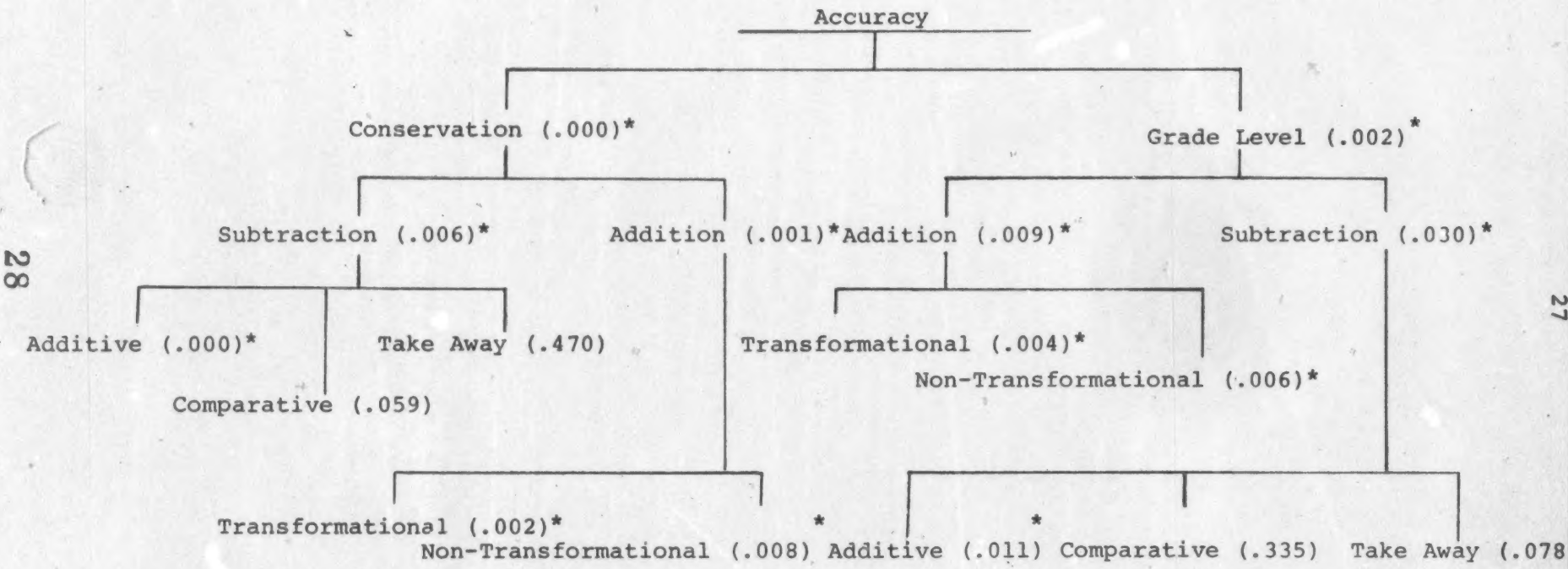
Insert Tables 5- 7About Here

subjects to perform the arithmetical operations accurately was significantly effected by both their ability to conserve (p = .00). It must be remembered that the subjects' ages (in months) and sex were treated as covariates in the analyses.

The effect of conservation upon accuracy was reflected in both the addition (p = .00) and subtraction (p = .01) accuracy scores. The significance of addition accuracy scores was found to be attributable to the conservation effect on both transformational (p = .00) and non-transformational (p = .01) problems. However, the conservation significant effect upon subtraction (p = .00) and, to a lessor degree, comparative subtraction (p = .06).

The significant effect of schooling upon accuracy was reflected in both addition (p = .01) and subtraction (p = .03) accuracy scores. The significance of schooling upon addition

Fig. 7 Summary of Analysis of Covariance of Accuracy Scores



* P < .05

Table 5

Summary of Analysis of Covariance Based on Total Accuracy Scores for Grade Level and Conservation

I.

Source						
Covariates	10.53	54.9627	2			
Grade Level	20.60	107.5556	1	107.5556	10.158	.002
Conservation	24.36	127.1363	1	127.1363	16.350	.000
Error	44.51	232.3454	67	4.7344		
Total	100.00	522.0000	71			

II.

Table of Means

Scores	K	1st	NC	C
$\bar{X}$ Accuracy Scores	4.44	6.89	4.15	6.54

TABLE 6

Summary of Analysis of Covariance Based Upon Addition and Subtraction Accuracy Scores for Grade Level and Conservation

I. Addition:

Source	%Var.	ss	df	ms	f	Sig.Level
Covariates	14.66	21.0411	2			
Grade Level	22.30	32.0000	1	32.0000	7.1841	.009
Conservation	18.04	25.8939	1	25.8939	11.2825	.001
Error	44.99	64.5650	67	1.4411		
Total	100.00	143.5000	71			

II. Subtraction:

Source	%Var.	ss	df	ms	f	Sig.Level
Covariates	3.21	8.0704	2			
Grade Level	8.84	22.2222	1	22.2222	4.933	.030
Conservation	15.22	38.2773	1	38.2773	8.109	.006
Error	72.74	182.9301	67	2.8011		
Total	100.00	251.5000	71			

III. Table of Means

Scores	k	l	nc	c
$\bar{X}$ Accuracy Scores; Addition	2.08	3.42	2.15	3.09
$\bar{X}$ Accuracy Scores; Subtraction	2.36	3.47	2.00	3.43

TABLE 7

Summary of Analysis of Covariance Based Upon Addition and Subtraction Accuracy Subscores for Grade Level and Conservation

I. Transformation Addition:

Source	%Var.	ss	df	ms	f	Sig.Level
Covariates	10.42	4.8837	2			
Grade Level	21.60	10.1250	1	10.1250	11.018	.022
Conservation	14.37	6.7369	1	6.7369	9.070	.004
Error	53.61	25.1294	67	.4838		
Total	100.00	46.8750	71			

II. Non-Transformation Addition:

Source	%Var.	ss	df	ms	f	Sig. Level
Covariates	13.09	5.7430	2	6.1250		
Grade Level	13.96	6.1250	1	6.1253	1.627	.006
Conservation	13.96	6.1253	1	.5079	7.392	.008
Error	58.99	25.8817	67			
Total	100.00	43.8750	71			

III. Additive Substraction:

Source	%Var.	ss	df	ms	f	Sig. Level
Covariates	3.56	1.9689	2			
Grade Level	7.26	4.0139	1	4.0139	6.769	.011
Conservation	17.63	9.7513	1	9.7514	15.332	.000
Error	71.56	39.5852	67	.5935		
Total	100.00					

TABLE 7, Continued

IV. Comparison Subtraction:

Source	%Var.	ss	df	ms	f	Sig. Level
Covariates	12.60	6.2962	2			
Grade Level	8.42	4.2094	1	4.2094	.9454	.335
Conservation	10.03	5.0139	1	5.0139	3.7053	.059
Error	69.95	34.4666	67	.5477		
Total	1000.00	49.9861	71			

V. Take Away Subtraction:

Source	%Var.	ss	df	ms	f	Sig. Level
Covariates	.00	.0224	2			
Grade Level	1.33	.5000	1	.5000	3.210	.078
Conversation	1.35	.5044	1	.5044	.528	.470
Error	97.26	36.4732	67	.6157		
Total	100.00	37.5000	71			

VI. Table of Means

Scores	k	1st	nc	c
$\bar{X}$ Accuracy Tr Add	.92	1.67	.96	1.48
$\bar{X}$ Accuracy NTr Add	1.17	1.75	1.19	1.61
$\bar{X}$ Accuracy Add Sub	.61	1.08	.46	1.06
$\bar{X}$ Accuracy Comp Sub	.25	.78	.15	.72
$\bar{X}$ Accuracy TA Sub	1.50	1.67	1.38	1.70

accuracy scores was attributable to both transformational ($p < .00$) and nontransformation ($p = .01$) problems. However, the schooling effect upon the subtraction abilities of the subjects is primarily attributable to their additive subtraction problem responses ($p = .01$).

In summary, the subjects' ability to model the mathematical problems is significantly effected by their ability to conserve numerousness. This particularly effects their ability to model additive and comparison subtraction problems. The subjects' ability to accurately solve the mathematical problems is significantly effected by their previous school experiences. Schooling effects both addition and subtraction accuracy, particularly transformational addition, nontransformational addition, and additive subtraction.

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